Random Decrement Technique for Modal Identification of Structures

S.R. Ibrahim*
Old Dominion University, Norfolk, Va.

An algorithm is developed to obtain the free responses of a structure from its random responses, due to some unknown or known random input or inputs, using the random decrement technique without changing time correlation between signals. The algorithm is tested using random responses from a "generalized payload" model and from the "space shuttle" model. The resulting free responses are then used to identify the modal characteristics of the two systems. The method is limited to structures that are linear or have small nonlinearities.

Introduction

IN general, the experimental identification of structural modes of vibration is carried out by measuring the input (or inputs) to the structure under test and the resulting responses due to this input. Some vibration testing techniques, in order to simplify the identification procedure, use the free responses of structures. In such cases, although the input excitation need not be measured, some initial excitation is applied to the structure, and free responses are measured immediately after the initial exciting force is removed.

There are situations where controlled excitation or initial excitation cannot be used. For example, if the structure to be tested is in operation, applying any kind of external force may cause undesirable interruption. Another example is the case of in-flight response measurements, where a complete knowledge of the excitation is usually not available. In such cases, the use of the "random decrement signature" technique (a special averaging procedure that is used to determine the step and/or impulse response from the random response) to obtain the free responses is a promising technique.

The random decrement signature technique[1] has been used successfully for failure detection and damping measurement of structures in single-station, single-mode response cases. Application of the random decrement signature technique to a multiple of signals changes the time correlation between the individual signals. If the resulting responses are to be used to identify several modes of a structure, the random decrement signature technique must be modified to keep the time correlation between signals unchanged.

In this paper, an algorithm is developed to obtain the free responses of a linear structure from its random responses, due to some unknown or known random input or inputs, using the random decrement technique without changing time correlation between signals. The algorithm is tested by applying it to random responses obtained from two real structures. The first structure is a generalized payload model previously tested using sine sweep method and analyzed by NASA Structural Analysis (NASTRAN). The second structure is the ⅛-scale space shuttle model with modal parameters previously determined using sine sweep method and fast Fourier transform (FFT). Only responses from four stations on the solid rocket boosters (SRB's) were considered in the case of the space shuttle model. The filtered random responses from these two structures were recorded and digitized. The free responses were then obtained from the digitized random responses using the modified random decrement technique. The resulting free responses were used as data for a time domain identification technique[2,3] to identify the modal parameters of these structures.

Received March 16, 1977; presented as Paper 77-368 at the AIAA/ASME 18th Structures, Structural Dynamics, and Materials Conference, San Diego, Calif., March 21-23, 1977 (in bound volume of Conference papers); revision received Aug. 1, 1977.

Index categories: Vibration; Structural Dynamics.

*Assistant Professor, Department of Mechanical Engineering and Mechanics. Member AIAA.

Background

A. Time Domain Identification Technique

This technique is described fully in Refs. 2 and 3. It uses the free responses (free decay) of a structure under test to identify its vibration parameters, namely, frequencies, dumping factors, and modal vectors in complex form. From the measured free responses of n stations on a structure under test, and assuming that the responses contain n modes, a matrix $[A]$ is formed such that

$$A = \begin{bmatrix} Y \\ Z \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}^{-1} \qquad (1)$$

where

$$x_{ij} = x_i(t_j)$$
$$y_{ij} = x_i(t_j + \Delta t) \qquad (i = 1,n;\ j = 1,\ 2n)$$
$$z_{ij} = x_i(t_j + 2\Delta t)$$

The eigenvectors of the A matrix are the modal vectors, and the eigenvalues α_i are related to the characteristic roots λ_i of the system through the equation

$$\alpha_i = e^{\lambda_i \Delta t} \qquad (2)$$

The practical application of the technique includes determining the number of modes in the response, adjusting the number of apparent stations, and, accordingly, adjusting the order of the mathematical model. Methods to reduce the effect of measurements noise are also described in Ref. 3.

Random Decrement Signature Technique

The basic concept of the "random decrement signature,"[1] is based on the fact that a random response of a structure due to a random input is composed of two parts: 1) deterministic part (impulse and/or step), and 2) random part (assumed to have a zero average). By averaging enough samples of the same random response, the random part of the response will average out, leaving the deterministic part of the response. To avoid averaging out the deterministic part of the signal, the samples can be taken starting always with 1) a constant level (this will give the free decay step response), 2) positive slope

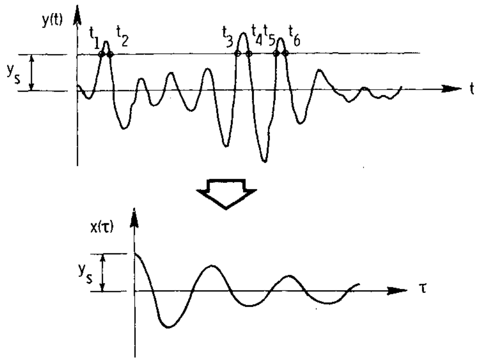

Fig. 1 One-station random and randomdec responses.

and zero level (this will give the free decay positive impulse response), and 3) negative slope and zero level (this will give the free decay negative impulse response). In Fig. 1, if $y(t)$ is the random response, the free decay response will be

$$x(\tau)=\frac{1}{N}\sum_{n=1}^{N}y(t_n+\tau) \tag{3}$$

with the condition $t_n=t$: $t_n=t$ when $y=y_s$ for case 1; $t_n=t$ when $y=0$ and $dy/dt>0$ for case 2; or $t_n=t$ when $y=0$ and $dy/dt<0$ for case 3.

Random Decrement Signatures and Modal Identification of Structures

The response resulting from applying the random decrement signature technique to a random response output of a structure under test is the free decay response. This response can be used in any identification technique (with force input equal to zero) to identify the natural frequencies and damping factors of the structure under test. This procedure has been widely used in flutter testing.[4,5] In flutter testing, determining natural frequencies and damping factors is the primary goal, and one response signal from the structure is sufficient to obtain such information.

For complete modal identification of a structure, mode shapes, frequencies, and damping factors of a structure under test are to be determined. This can be done by using a random input and the random responses of the structure at the stations of interest due to that input in some identification technique. If complete knowledge of the random input to the structure is not available, the use of random decrement signature technique seems to be the answer.

Applying the random decrement signature technique to each individual signal of the multiple of signals simultaneously recorded will change the time correlation between these individual signals. Using the resulting signatures to identify the structure will give the correct frequencies and damping factors but will give erroneous mode shapes because of the change in the time correlation between the individual signals

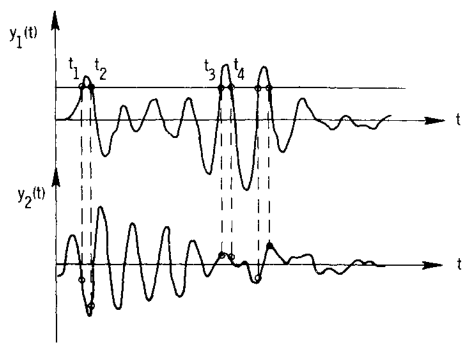

Fig. 2 Two-station random response.

Multiple-Signal Random Decrement Technique

In this section, an algorithm will be developed to obtain the free decay responses of a multiple of random response signals simultaneously recorded from a structure using the random decrement technique without changing the time correlation between signals. To derive and prove this algorithm, a two-station response will be used, and then the algorithm can be generalized to any number of stations.

In Fig. 2, if $y_1(t)$ and $y_2(t)$ are the random responses of a structure at stations 1 and 2 due to some known or unknown random input, a free decay response of these two stations can be written as

$$\begin{bmatrix} x_1(\tau) \\ x_2(\tau) \end{bmatrix}=\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix} y_1(t_i+\tau) \\ y_2(t_i+\tau) \end{bmatrix} \tag{4}$$

with condition

$$t_i=t \text{ when } y_1(t)=y_s \tag{5}$$

The condition imposed by Eq. (5) implies that the free decay response of station 1 will have initial conditions of $x_1(0)=y_s$ and $(dx_1/d\tau)_{\tau=0}=0$. This means that the free decay response of station 1 resulting from the random decrement averaging process cannot diminish by the possibility of being averaged out.

Unlike station 1, the random response of station 2 has no condition on the start of the samples. Averaging of station 2 samples exactly follows that of station 1.

The question now is, is it possible for the deterministic response of station 2 to be averaged out, since no condition was applied to the start of its samples? The answer to this question is readily available by examining Eqs. (4) and (5). Since the free decay of station 1 exists, and since the two stations' random responses were recorded simultaneously on the same structure and these two stations are dynamically coupled, it is impossible to have response from one station and no response from the other.

We state the algorithm for a multiple of signals as follows. If $y(t)$ is the random response vector of n stations on a structure due to some known or unknown random input or inputs with zero mean, the free decay response vector for these n stations can be written as

$$x(\tau)=\frac{1}{N}\sum_{i=1}^{N}y(t_i+\tau) \tag{6}$$

with one of the following conditions:

$$t=t_i \quad \text{when} \quad y_l(t)=y_s=\text{(constant level)}$$

$$\text{or} \quad y_l(t)=0 \quad \text{and } \dot{y}_l(t)>0$$

$$\text{or} \quad y_l(t)=0 \quad \text{and } \dot{y}_l(t)<0$$

where l is any arbitrary leading station of the n stations, and N is the number of averages.

Multiple-Signal Randomdec Computation

Assuming that $y_{i,j}$ ($i=1,2,...n$ and $j=1,2,...m$) is the random response recorded simultaneously from n stations on a structure and m is number of data points for every station, any of these stations can be assigned as the leading station. This is completely arbitrary as long as all of the stations are recorded simultaneously. The randomdec response $x_{i,k}$ according to Eq. (6) will be

$$x_{i,k}=\frac{1}{N}\sum_{r=1}^{N}y_{i,(r+k)}$$

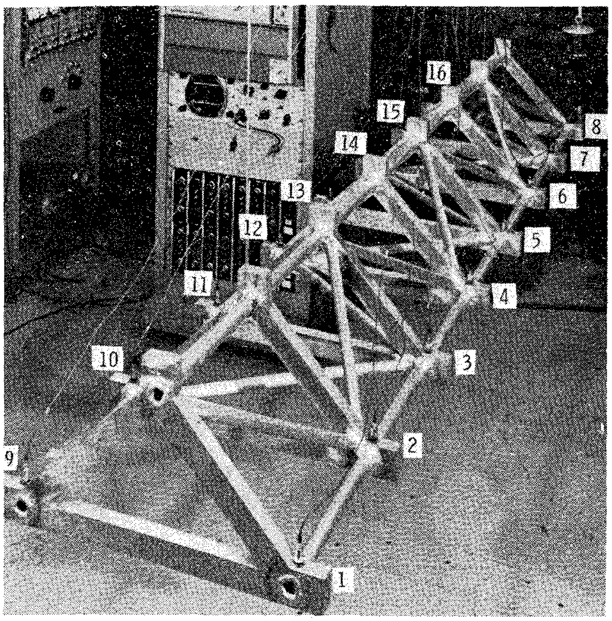

Fig. 3 Payload model.

where $r=1,2...N$ designates zero crossings with positive slopes of the leading station response and $k=1,...M$. N will be the total number of averages, and M will be the number of randomdec data points for each station.

Experimental Results

A. Payload Model

Figure 3 shows the payload model structure. The model was analyzed previously by NASTRAN and tested using a sine sweep test. The model was tested using the time domain technique both with and without the random decrement analysis. Sixteen accelerometers were fixed to the eight bulkheads, and their responses were recorded on magnetic tape in two data groups. The first data group contained accelerometers 1-8 on one side of the model, and the second data group contained accelerometers 9-16 on the other side, and also accelerometer 8, to relate the two data groups.

A random input was applied at station 8. The experiment was designed so that a double switch can cut off the random input and at the same time generates a step signal to be recorded on a separate channel, indicating the start of the free decay. The random part and the free decay part of the responses of data group 1 were recorded. The same procedure was repeated for data group 2. The free decay part of the response was used as data for the time domain technique. Technique and results are reported in Ref. 3.

To test the algorithm developed in this paper, the random responses (filtered above 500 Hz) of the 17 stations were digitized and analyzed by a computer program to obtain free decay responses using the multiple-signal random decrement technique described by Eq. (6). It is to be noted here that, since the responses from the 17 stations were not recorded simultaneously, a leading station has to be used for each data group. In this experiment, station 1 was used as a leading station for stations 1-8, and station 9 was the leading station for stations 9-16 and station 8. Figures 4a and 4b show the random response of stations 2 and 10. Figures 4c and 4d show their calculated free decay.

To illustrate the importance of using a leading station from the same run, the free decay response of station 10 (group 2) calculated with station 1 (from group 1) as leading station is shown in Fig. 4e. Comparing Figs. 4d and 4e, it is clear that the level of the free decay response in Fig. 4e is much lower than that of Fig. 4d.

The number of samples averaged to obtain the free decay response from the random response was 753. All samples were chosen such that the response of the leading station starts with zero level and positive slope. Table 1 shows the natural frequencies and damping factors obtained from the time domain technique using the recorded free decay responses and using calculated randomdec responses. Also shown are frequencies from NASTRAN analysis.[6] Table 2 shows a comparison of the identified modal shapes of the 74.15-Hz mode (first bending), one obtained from recorded free decay data and the other from calculated randomdec data. Comparison is excellent, considering that the recorded free decay data had 22% noise in them and that the random responses had about 10% saturated point. Another factor that contributed to the discrepancy between the two modes in comparison took place during the analog-to-digital conversion of the random responses of the two data groups. Signals simultaneously recorded should be simultaneously digitized. In this experiment, responses were digitized in four groups: (1, 3, 5, 7), (2, 4, 6, 8), (9, 11, 13, 15, 17), and (10, 12, 14, 16). If this regrouping has to be used, one station has to be common between each two subgroups. In this experiment, the regrouping should have been (1, 3, 5, 7), (1, 2, 4, 6, 8), (9, 11, 13, 15, 17), and (9, 10, 12, 14, 16).

Table 1 Identified frequencies and damping factors for payload model

Mode No.	Time Domain With Recorded Free Decay Responses		Domain With Calculated Randomdec Responses		NASTRAN
	Frequency (Hz)	Damping Factor	Frequency (Hz)	Damping Factor	Frequency (Hz)
1	74.15	0.0019	74.15	0.0029	73.4
2	78.74	0.0017	78.75	0.0010	80.1
3	119.83	0.0013	120.27	0.0010	117.3
4	156.63	0.0007	156.61	0.0013	158.9
5	161.93	0.0007	161.77	0.0007	159.9
6	216.44	0.0013	216.47	0.0010	218.6
7	245.18	0.0034	245.00	0.0036	244.6
8	259.30	0.0007	260.70	0.0004	253.1
9	280.94	0.0018	280.75	0.0017	283.0
10	325.31	0.0005	325.30	0.0002	----

Table 2 An identified modal vector for payload model (frequency = 74.14 Hz)

Station	With Recorded Free Decay Responses		With Calculated Randomdec Responses	
	Amplitude	Phase (degrees)	Amplitude	Phase (degrees)
1	83.37	-6.3	86.79	-2.6
2	18.02	-13.2	20.10	-2.2
3	39.86	179.2	41.43	177.8
4	72.58	178.7	75.7	179.1
5	71.58	174.4	74.46	177.6
6	30.77	176.0	35.13	180.5
7	31.43	1.2	31.63	-2.4
8	100.00	0.0	100.00	0.0
9	99.34	-1.4	96.02	-14.0
10	34.69	7.1	28.26	-1.7
11	28.82	164.3	35.05	160.0
12	68.58	176.4	71.62	175.6
13	71.63	175.8	71.15	164.1
14	39.92	182.12	38.54	178.2
15	15.43	-27.00	23.44	-21.9
16	73.00	-9.8	84.35	-5.1

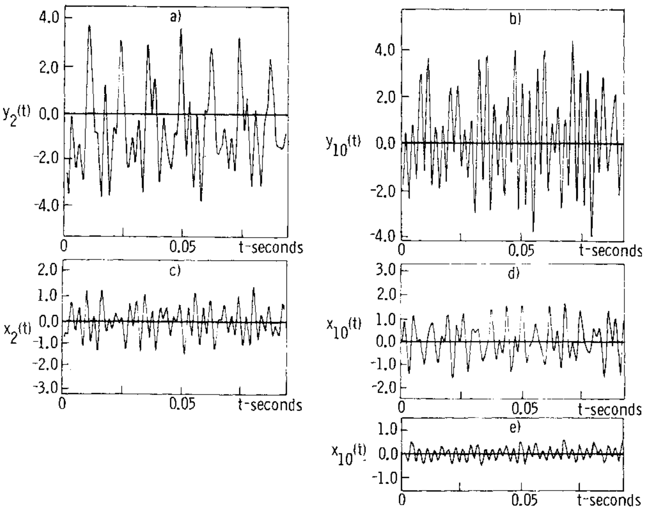

Fig. 4 Random and randomdec free decay response of selected stations on the payload model.

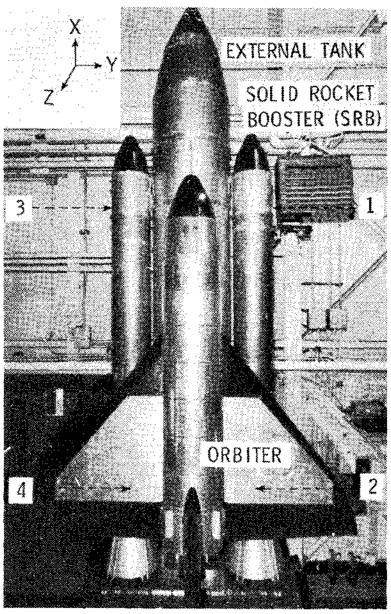

Fig. 5 The ⅛-scale space shuttle model.

B. The ⅛-Scale Space Shuttle Model

The ⅛-scale "space shuttle" model is shown in Fig. 5. Modal survey testing of the model by FFT[7] has been carried out at NASA Langley Research Center as part of the Space Shuttle Program. To test the algorithm developed in this paper, the random responses of four stations on the two "solid rocket boosters" (Fig. 5) in the Y direction already recorded on analog tape were digitized at a rate of 500 samples per second. These four random responses were then used to calculate the randomdec free decay responses using the multiple-signal random decrement technique [Eq. (6)]. The resulting free responses were then used as data for the time domain identification technique to identify the vibration modal characteristics of these four stations.

Figures 6a and 6b show the random responses of stations 1 and 4. The free decay responses resulting from samples average are shown in Figs. 6c and 6d. All samples of station 1, as a leading station, were chosen to start with zero levels and positive slopes. Frequencies identified by the random decrement/time domain technique are listed in Table 3, together with the FFT frequencies. It is to be noted that all of the four stations considered in this experiment were on the two SRB's and in the Y direction.

Table 3 Shuttle model identified frequencies

FFT	Randomdec/Time Domain	Remarks
13.8	----	Gear-Train Rotation
16.5	16.57	
17.6	18.17	
20.5	20.71	
21.6	----	ORB. Roll Mode
24.3	----	ET 1st Torsion
---	25.50	
26.0	26.29	
---	26.69	
28.2	----	ORB. Longitudinal
30.0	----	
32.2	32.10	
34.5	----	
---	36.36	
28.0	38.25	
43.2	43.01	
---	45.00	
47.5	----	
48.5	----	ET Z-Direction Bending
51.0	50.2	
57.5	58.05	
58.5	59.2	
---	60.07	
---	61.68	
69.0	68.65	
---	71.55	
76.3	76.23	
---	76.81	
---	76.85	

Table 4 An identified shuttle model mode

Sta.	FFT		Time Domain	
	Relative Amplitude	Phase (degrees)	Relative Amplitude	Phase (degrees)
1	100.00	0.0	100.00	0.0
2	55.00	0.4	61.65	0.98
3	86.46	19.1	98.77	22.3
4	54.07	18.7	33.96	15.4

Comparison of modal shapes of the 76.3-Hz FFT and the 7623-Hz "time domain" modes (assuming that they are the same mode) is shown in Table 4. It is to be noted that the time domain technique identified modes with frequencies very close to the 76.23-Hz mode (76.81 and 76.85), whereas FFT did not show these modes. If the difference in the two modal vectors in comparison is not because of different gains or different data processing, the FFT modal vector might have contained contributions from the two very close modes.

Conclusions

The "multiple-signal random decrement technique" described in this paper makes it possible to calculate the free decay response of a multiple of random response signals simultaneously recorded from a structure under test. The resulting free decay response can be used to identify the vibration modal characteristics of structures without altering phase relations between individual stations. This approach is extremely useful when complete knowledge of the random

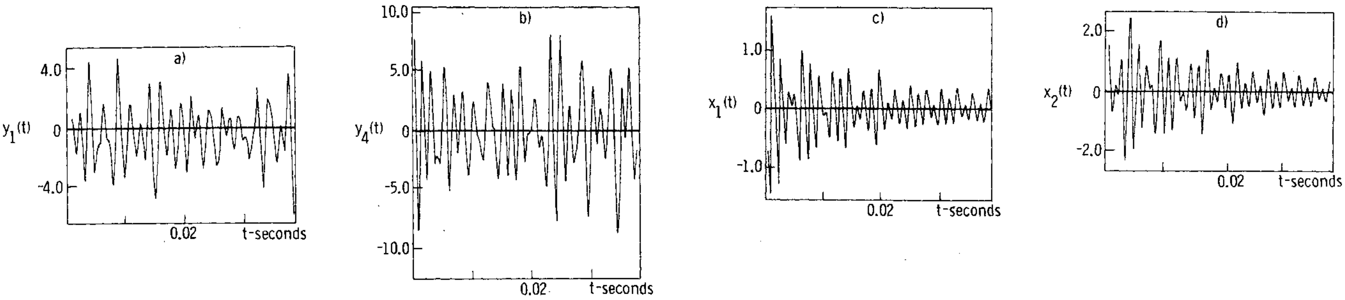

Fig. 6 Random and randomdec free decay response of selected stations on the shuttle model.

input to the structure under test is not available. It is also useful for identification techniques that use free decay data.

Acknowledgments

This work is supported by NASA Langley Reserch Center under Grant No. NSG 1325 monitored by B.R. Hanks. The author would like to expresss his gratitude to all personnel of the Vibration Section at NASA Langley Research Center. Special thanks are due to R.W. Herr, S.A. Leadbetter, and U.J. Blanchard for allowing me to use their unpublished data from the Payload Dynamics and Space Shuttle Programs.

References

[1] Cole, H.A. Jr., "On-Line Failure Detection and Damping Measurement of Aerospace Structures by Random Decrement Signatures," NASA CR-2205, March 1973.

[2] Ibrahim, S.R. and Mikulcik, E.C., "The Experimental Determination of Vibration Parameters from Time Responses," *Shock and Vibration Bulletin,* Bull. 46, part 5, Aug. 1976.

[3] Ibrahim, S.R. and Mikulcik, E.C., "A Method for The Direct Identification of Vibration Parameters from the Free Responses," *Shock and Vibration Bulletin,* Bull. 47, part 4, Sept. 1977.

[4] Chang, C.S., "Study of Dynamic Characteristics of Aeroelastic Systems Utilizing Randomdec Signatures," NASA CR 132563, 1975.

[5] Hammond, C.E. and Doggett, R.V. Jr., "Determination of Subcritical Damping by Moving-Block/Randomdec Applications," *Proceedings of the NASA Symposium on Flutter Testing Techniques,* Oct. 9-10, 1975.

[6] Herr, R.W., "Payload Dynamics Program," unpublished data, NASA Langley Research Center, 1975.

[7] Leadbetter, S.A., "1/8 Scale Space Shuttle Dynamic Testing," unpublished data, NASA Langley Research Center, 1976.